

Discounting and climate change: a non-marginal policy choice

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Abstract

Climate change raises a number of complex issues for economics, of which discounting is one of the most prominent. Besides familiar arguments over, for example, what is the appropriate rate at which to discount utility, the conventional toolkit of cost-benefit analysis might be too simplistic to handle the key features of climate change. This toolkit relies on the assumption that projects to be evaluated are marginal perturbations around an exogenous future growth path. Hence the shadow discount rate can be calibrated on this path, be it certain or uncertain. However, the impacts of climate change could be large and irreversible, such that policy choices today over reducing greenhouse gas emissions could be more than just marginal perturbations. They could lead to non-marginal shifts in the future growth path. In this paper we explore these ideas theoretically and empirically. We demonstrate in theory that policies to reduce greenhouse gas emissions can fail a conventional cost-benefit analysis, when in fact social welfare is increased, and vice versa. In addition, we use an integrated assessment model to identify some plausible empirical instances in which this occurs. In the welfare economics of climate change, shortcuts can be misleading.

Keywords: Discounting, climate change, cost-benefit analysis

JEL classification: D61, D81, D90, H43, Q20, Q40

¹ At present all opinions expressed, and errors committed, are due to Simon Dietz.

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1. Introduction

Because the climate-change externality occurs with a lag but thereafter persists for a very long time, the benefits of avoided climate change accrue decades and centuries after many of the costs are paid. Thus intertemporal aggregation is one of the most important issues for the economics of climate change, and one of the most disputed (see Portney and Weyant, 1999). Often, disputes turn on what is an appropriate rate at which to discount *consumption* over the very long run, in large part because the consumption discount rate can be estimated in a number of markets (but see Dietz *et al.*, 2008, and Stern, 2008, on the difficulties in doing so). Discounting at a comparatively high rate will reduce the future benefits of avoided climate change to the extent that even fairly significant climate-change damage in the far-off future will be worth absorbing in the process of development. This is fairly easy to see: for example, one dollar received in two hundred years' time is worth roughly one half of one thousandth of one cent today when discounted exponentially at five percent.

However, it is important to 'unpack' the consumption or shadow discount rate to consider its variables and parameters. Behind the shadow discount rate in most analyses is the parsimonious 'workhorse' model of standard welfare economics, in which social welfare is additive over the utilities of individuals in different time periods, sometimes in different regions and sometimes in different states of the world. The utility function is typically isoelastic and utility depends on the aggregate consumption of goods and services. The costs of climate change and of emissions reductions are deducted from aggregate consumption in an additively separable framework.² This places a great deal of importance on two parameters; the utility discount rate and the elasticity of social marginal utility of consumption, the latter of which becomes a measure of social inequality aversion over space and time, and relative risk aversion. Indeed, the recent dispute about the formal welfare economics of the *Stern Review* (Stern, 2007) has demonstrated their importance.³

Uncertainties are associated with every step in the chain that links long-run trends in population, economic growth and technology, through the entire process of anthropogenic climate change, to impacts on the natural environment, economies and societies. Moreover it is generally considered that, insofar as one can describe a probability distribution over the impacts of climate change, it has a

² On why this may miss crucial aspects of the problem, see Hoel and Sterner (2007) and Sterner and Persson (in press).

³ For critical perspectives see Beckerman and Hepburn (2007), Dasgupta (2006), Nordhaus (2007) and Weitzman (2007). *Stern Review* authors responded in Dietz *et al.* (2007a and b).

positive skew, perhaps very large. Examples of climatic changes that might produce impacts in the longer right-hand tail of the distribution include a significant slow-down in the thermohaline ocean circulation and large-scale melting of the Greenland and West Antarctic ice sheets (Lenton *et al.*, 2008; Schneider and Lane, 2006). Thus aggregation across states of the world may be no less important to the economics of climate change than intertemporal aggregation (Weitzman, 2007, 2008).

In this paper we focus on a different aspect of the discounting problem, namely the variable determining social welfare and the discount rate – consumption. Here, the issues of aggregation over time and over states of the world come together. Our basic point is that the shadow discount rate depends on the future path of consumption. For every path there will be a different shadow discount rate. Moreover, because the impacts of climate change could be large and irreversible, there could be a large difference between the future paths of consumption under ‘business-as-usual’ climate change compared with those if greenhouse gas emissions are reduced. Both of these paths could also be very different from an extrapolation of past trends in consumption, which neither laboured under climate change⁴ nor under emissions reductions. In such a situation we may not be able to rely on the conventional simplifications of project appraisal, which rely on changes that are marginal with respect to the whole growth path.

In recent years, economists have developed analyses that account for exogenous uncertainty in future consumption. These have given rise to the idea that uncertainty over future consumption can be tackled via the application of an exogenous, certainty-equivalent, time-declining discount rate (e.g. Gollier, 2001, 2002a, b; Groom *et al.*, 2005; Newell and Pizer, 2003; Weitzman, 1998, 2004). The difficulty is that this approach depends on the assumption that uncertainty over future consumption is independent of the policy choice being contemplated. This may not be the full story here, because the policy decision in question may itself lead to large differences in consumption paths. Hence we are faced with endogenous uncertainty, as well as exogenous uncertainty.

Instead we have to go back to some extent to the underlying objective against which consumption paths are evaluated over time (and from which discount rates are derived); generally an integral of utility of consumption. This is not, of course, to abandon a basic cost-benefit calculus but simply to avoid using an often helpful short-cut, the discount rate, which is not valid in this context of non-

⁴ At least in the sense of the modern phenomenon of climate change. But consumption has in the past been influenced by climatic variability (e.g. the ‘Dust Bowl’).

marginal changes. Section two rehearses the theory of non-marginal policy choices and their implications for simplified methods of project appraisal. Section three describes the integrated assessment model that we use, PAGE. Section four outlines our empirical results and section five concludes.

2. Theory

Let utility per capita u be a smooth and continuously differentiable functional of consumption per capita c for a path of consumption i at time t :

$$u_t^i = u(c_t^i) \quad (1)$$

Total social welfare V along consumption path i is the integral of per capita utility over all time periods along consumption path i :

$$V^i = \int_{t=0}^{\infty} u(c_t^i) e^{-\rho_t t} dt \quad (2)$$

where ρ is the utility discount rate and $\rho > 0$ over an infinite horizon to ensure the integral converges (in a finite time period $\rho \geq 0$). It is worth noting that almost all previous empirical work has considered a finite time period: no integrated assessment model has an infinite horizon. Accordingly, in our theoretical discussion we generalise over an infinite horizon, while in our empirical analysis we work in finite time. In this theoretical section, we also discuss a deterministic, global setting for simplicity. In a stochastic setting with many regions we would integrate over states of the world and regions respectively, as we do in our empirical analysis. This brings the form and parameters of the utility function strongly into play, as we explain in the results section.

From (2) it follows that the shadow discount factor λ for path i at time t is:

$$\lambda_t^i = \frac{\partial}{\partial c} V^i = u'(c_t^i) e^{-\rho_t t} \quad (3)$$

By indexing λ to the path of future per capita consumption i we create a general framework for the evaluation of projects. A project is itself represented as a change in the path of consumption per capita, denoted Δi , where j indicates that the project can either be 'small' ($j=s$) or 'large' ($j=l$) in the sense of being marginal or non-marginal respectively, or in other words producing partial or general equilibrium effects respectively.

From (3), the shadow (i.e. consumption) discount rate r for path i at time t is:

$$r_t^i = \frac{-\dot{\lambda}_t^i}{\lambda_t^i} = -\frac{d}{dt} \log u'(c_t^i) + \rho_t \quad (4)$$

For an isoelastic utility function of usual form

$$u(c_t^i) = \frac{c_t^{i^{1-\eta}} + 1}{1-\eta} \quad (5)$$

where η is the elasticity of social marginal utility of consumption, then the shadow (consumption) discount rate may be rewritten:

$$r_t^i = \eta \frac{\dot{c}_t^i}{c_t^i} + \rho_t \quad (6)$$

The theory of cost-benefit analysis for marginal projects is represented by figure 1. If the project Δi is small (which we denote as $\Delta^s i$), and more specifically is marginal relative to the future path of consumption per capita, the project can therefore be evaluated at the set of shadow discount factors on the path $u(c_t^i)$, where c_t^i is an exogenous estimate of future consumption per capita and can be uncertain (in which case we would take the expectation), but must not be affected by $\Delta^s i$.

Proposition 1

Given the marginal investment $\Delta^s i$:

$$\int_0^{\infty} \exp\left(\int_0^t -r_t^i dt\right) \Delta_t^s dt > 0 \quad (7)$$

$$\Rightarrow \int_0^{\infty} u(c_t^i + \Delta_t^s) e^{-\rho_t t} dt > \int_0^{\infty} u(c_t^i) e^{-\rho_t t} dt \quad (8)$$

Proof

Substitute the right-hand side of (4) into the left-hand side of (7):

$$\int_0^{\infty} \exp\left(\int_0^t -r_t^i dt\right) \Delta_t^s dt = e^{-\rho_t t} \exp\left(\int_0^t \frac{d}{dt} \log u'(c_t^i) dt\right) \Delta_t^s dt \quad (9)$$

$$= e^{-\rho_t t} \exp\left(\log \frac{u'(c_t^i)}{u'(c_0^i)}\right) \Delta_t^s dt \quad (10)$$

$$= \frac{1}{u'(c_0^i)} \int_0^{\infty} e^{-\rho_t t} u'(c_t^i) \Delta_t^s dt \quad (11)$$

For a marginal investment, Δ_t^s , the following Taylor approximation holds:

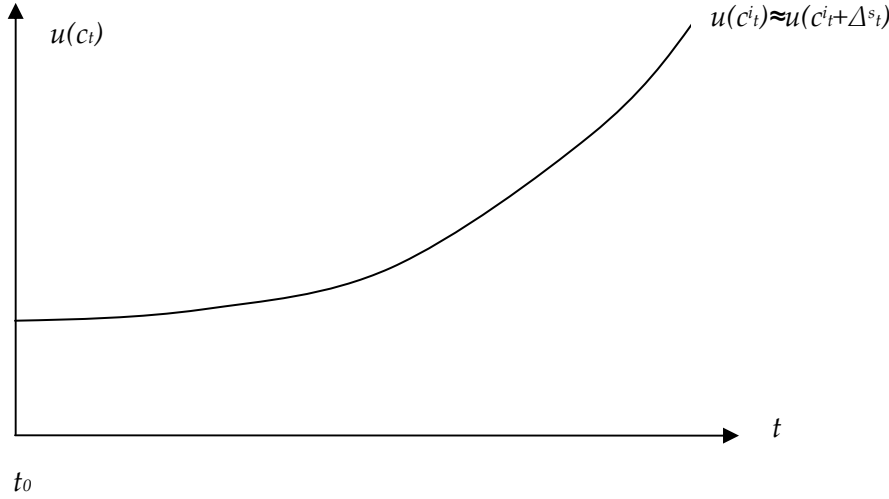
$$u(c_t^i + \Delta_t^s) \approx u(c_t^i) + \Delta_t^s u'(c_t^i) \quad (12)$$

Substitute (12) into (11) and then into (7):

$$\int_0^{\infty} e^{-\rho t} [u(c_t^i + \Delta_t^s) - u(c_t^i)] dt > 0 \quad (13)$$

From which it follows that (8) holds. ■

Figure 1. Project Δ^s_t is a marginal perturbation around the path of future per capita consumption.

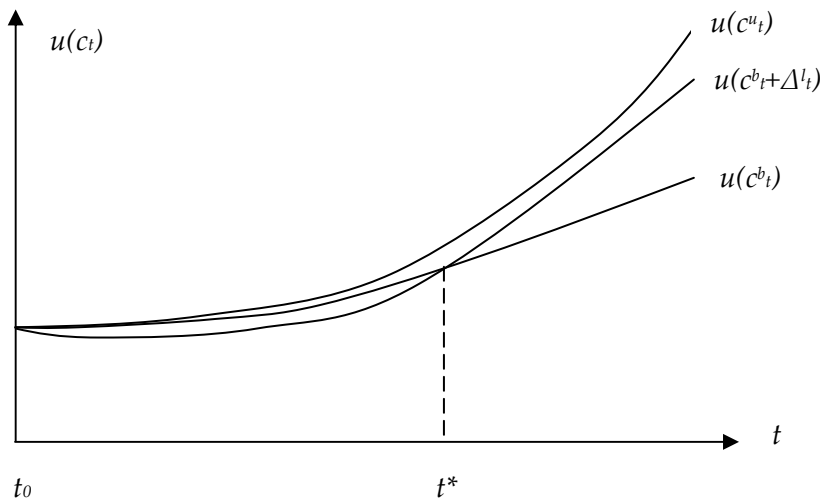


For projects large enough to have non-marginal effects on future per capita consumption, figure 2 attempts to present the issues intuitively, introducing concepts from climate-change policy. It plots three paths for the future growth of global utility per capita. Path $u(c^b_t)$ represents global utility per capita on a path where greenhouse gas emissions are uncontrolled ($i=b$ where b stands for ‘business-as-usual’), resulting in climate change that has a non-marginal effect on consumption per capita and hence utility per capita both through the costs of adaptation and especially through residual impacts (we could specify utility as a direct function of environmental quality and hence climate change without loss of generality). Path $u(c^b_t + \Delta^l_t)$ represents utility per capita along a path where greenhouse gas emissions are controlled. The ‘project’ Δ^l_t is a non-marginal global investment in emissions reductions, because it avoids many of the costs of climate change and perhaps also because the cost of emissions reductions is itself non-marginal.⁵ Path $u(c^u_t)$ is a counterfactual, in which greenhouse gas emissions are uncontrolled but there are no damages from climate change ($i=u$ where u stands for ‘utopia’). This would normally be estimated on past trends, since we are yet to experience non-marginal impacts of climate change (if indeed there are any to come in the future) and similarly we are yet to make non-marginal investments in reducing greenhouse gas emissions (if there are any). Proposition 1 indicates that path $u(c^u_t)$ is the appropriate basis for a set of shadow discount factors for climate-change policy, if one assumes that climate change has a marginal effect on the future path of per capita consumption and in turn that the investment to reduce emissions is also marginal. Many theoretical and empirical discussions of discounting in the context of climate change do in fact take

⁵ It could be the case that only the cost of emissions reductions is non-marginal, in which case we would draw $u(c^b_t + \Delta^l_t)$ below both $u(c^b_t)$ and $u(c^u_t)$ over all time (the latter two being equal). However this is a trivial case for our purposes, because the welfare change as a result of the project will always be negative.

$u(c^u_t)$ to be the appropriate long-run path, along which to discount the net benefits of emissions reductions. On the other hand, if the assumption that the project is marginal does not hold, it will not be. If $u(c^u_t)$ is taken to be uncertain, the expected shadow discount factors estimated on past trends (or by some other means such as expert forecast) may be quite similar to those sets on $u(c^b_t)$ or $u(c^b_t + \Delta^l_t)$. Weitzman (2007) has made this argument in his review of the *Stern Review*. But it may not and critically depends on how the uncertainty is estimated and to what extent it (feliciously) approximates the effects of climate change and of emissions reductions. Moreover it is less likely to be a good approximation of both $u(c^b_t)$ and $u(c^b_t + \Delta^l_t)$.

Figure 2. Benefits of investment Δ^l_t are less than costs when $t < t^*$, but exceed costs when $t > t^*$.



We want to examine the circumstances in which proposition 1 gives a misleading signal of the welfare consequences of undertaking the investment in emissions reductions Δ^l_t . The project Δ^l_t is non-marginal so the difference between the paths of per capita utility with and without it will be, by definition, large. It follows that both these paths will also be different to the 'utopian' path. We cannot simplify using equation (7) and must instead measure the difference between social welfare on the path corresponding to the investment in emissions reductions – the path $u(c^b_t + \Delta^l_t)$ in figure 2 – and social welfare on the business-as-usual path $u(c^b_t)$, where the former must be strictly greater than the latter. Alternatively we could measure the difference between the change in welfare with and without the project, where the change with the project should be strictly smaller than without. This change is due to the change in per capita consumption arising from the sum of the costs of climate change and the cost of emissions reductions. The change in per capita consumption along the path $u(c^b_t + \Delta^l_t)$ should be evaluated at the discounted marginal utility of consumption on that same path, in other words with

the set of shadow discount factors $\lambda^{b+\Delta l_t}$. Similarly, the change in per capita consumption along the path without the project $u(c^b_t)$ should be evaluated at the discounted marginal utility of consumption commensurate with $u(c^b_t)$, which is the set of shadow discount factors λ^b_t . An error will be introduced by using the shortcut in equation (7), because it in effect discounts changes in per capita consumption with and without the project using the common set of shadow discount factors λ^i_t . This set cannot simultaneously be consistent both with the path $u(c^{b_t+\Delta l_t})$ and with the path $u(c^b_t)$. The ‘business-as-usual’ set of shadow discount factors λ^b_t corresponds to $u(c^b_t)$, but it will not correspond to $u(c^{b_t+\Delta l_t})$. The ‘utopian’ set of shadow discount factors λ^u_t will correspond neither to $u(c^b_t)$ nor to $u(c^{b_t+\Delta l_t})$.

In figure 2, the investment Δl_t reduces utility relative to $u(c^b_t)$ for an initial time period. The cost of emissions reductions initially exceeds the benefits due to the slow response of the stock externality. However, at some point $t>0$, the costs of climate change avoided by Δl_t begin to exceed the cost of emissions reductions, increasing utility relative to $u(c^b_t)$ for the remaining period. $u(c^b_t)$ and $u(c^{b_t+\Delta l_t})$ are always below $u(c^u_t)$, since both climate change and emissions reductions are costly. From figure 2 we can see that $u(c^u_t) > u(c^b_t) \geq u(c^b_t + \Delta l_t)$ for all t , $0 < t \leq t^*$. But $u(c^u_t) > u(c^b_t + \Delta l_t) \geq u(c^b_t)$ for $t \geq t^*$.

Proposition 2a

$$\text{If } \int_0^\infty \lambda_t^u \Delta_t^l dt < 0 \quad (14)$$

$$\Rightarrow \int_0^\infty u(c_t^b + \Delta_t^l) e^{-\rho_t t} dt \leq \int_0^\infty u(c_t^b) e^{-\rho_t t} dt \text{ or } \int_0^\infty u(c_t^b + \Delta_t^l) e^{-\rho_t t} dt \geq \int_0^\infty u(c_t^b) e^{-\rho_t t} dt \quad (15)$$

$$\text{Similarly if } \int_0^\infty \lambda_t^u \Delta_t^l dt > 0 \quad (16)$$

Then (15).

Proof

Assuming $u'(c_t^i) > 0$, $u''(c_t^i) < 0$, then $\lambda_t^u < \lambda_t^b \leq \lambda_t^{b+\Delta l}$ for $0 < t \leq t^*$. But then $\lambda_t^u < \lambda_t^{b+\Delta l} \leq \lambda_t^b$ for $t \geq t^*$.

Figure 2 shows that $u(c^u_t)$ lies above $u(c^b_t + \Delta^l_t)$ and $u(c^b_t)$ for all $t, t > 0$. Assuming the utility function is strictly concave, the set of shadow discount factors λ^u_t always lies below the corresponding sets $\lambda^{b+\Delta^l_t}$ and λ^b_t . The bias arising from employing equation (7) with the set of shadow discount factors λ^u_t is therefore in the direction of underestimating welfare changes both on the path $u(c^b_t + \Delta^l_t)$ and on the path $u(c^b_t)$ for all $t, t > 0$. When $0 < t < t^*$, welfare changes on $u(c^b_t + \Delta^l_t)$ are underestimated more than welfare changes on $u(c^b_t)$, since the difference between $u(c^u_t)$ and $u(c^b_t + \Delta^l_t)$ is greater than the difference between $u(c^u_t)$ and $u(c^b_t)$. In other words:

$$\lambda^{b+\Delta^l}_t - \lambda^u_t > \lambda^b_t - \lambda^u_t \quad (17)$$

Thus for $0 < t < t^*$, the investment Δ^l_t yields net costs and these costs are underestimated. Conversely when $t > t^*$, welfare changes on $u(c^b_t + \Delta^l_t)$ are underestimated less than welfare changes on $u(c^b_t)$, since the difference between $u(c^u_t)$ and $u(c^b_t)$ is the greater:

$$\lambda^b_t - \lambda^u_t > \lambda^{b+\Delta^l}_t - \lambda^u_t \quad (18)$$

For $t > t^*$, the investment Δ^l_t yields net benefits; these benefits are also underestimated. Thus, as proposition 2a tells us, if Δ^l_t fails discounted with λ^u_t , underlying social welfare may be either lower, equal or higher after having undertaken the project. The same is true if Δ^l_t passes discounted with λ^u_t . The sign and size of the bias depends on by how much the net costs of Δ^l_t are underestimated for $0 < t < t^*$, compared with by how much the net benefits of Δ^l_t are underestimated for $t > t^*$. The more the latter (former) dominates, the more likely it is that Δ^l_t will fail when in fact social welfare is higher. ■

Proposition 2b

$$\text{If } \int_0^\infty \lambda^b_t \Delta^l_t dt < 0 \quad (19)$$

Then (15).

$$\text{Similarly if } \int_0^{\infty} \lambda_t^b \Delta_t^l dt > 0 \quad (20)$$

Then (15).

Proof

$u(c_{b_t}^b)$ lies above $u(c_{b_t}^b + \Delta_t^l)$ for all t , $0 < t < t^*$. Again assuming the utility function is strictly concave, the set of shadow discount factors $\lambda_{b_t}^b$ lies below the corresponding set $\lambda_{b_t + \Delta_t^l}^b$. Employing equation (7) and this time discounting with $\lambda_{b_t}^b$, welfare changes on $u(c_{b_t}^b)$ will be correctly estimated, but welfare changes on $u(c_{b_t}^b + \Delta_t^l)$ will be underestimated. Thus for $0 < t < t^*$, the investment Δ_t^l yields net costs and these costs are underestimated. For $t > t^*$, $u(c_{b_t}^b)$ lies below $u(c_{b_t}^b + \Delta_t^l)$, so $\lambda_{b_t}^b$ lies above the corresponding set $\lambda_{b_t + \Delta_t^l}^b$. Welfare changes on $u(c_{b_t}^b)$ will be correctly estimated, but welfare changes on $u(c_{b_t}^b + \Delta_t^l)$ will be overestimated. For $t > t^*$, the investment Δ_t^l yields net benefits and these benefits, being the difference between the welfare equivalent of the total costs of climate change and emissions reductions on $u(c_{b_t}^b)$ and on $u(c_{b_t}^b + \Delta_t^l)$, are once again underestimated. Like proposition 2a, 2b tells us that if Δ_t^l fails discounted with $\lambda_{b_t}^b$, underlying social welfare may be either lower, equal or higher after undertaking the project. The same is true if Δ_t^l passes discounted with $\lambda_{b_t}^b$. The sign and size of the bias depends on by how much the net costs of Δ_t^l are underestimated for $0 < t < t^*$, compared with by how much the net benefits of Δ_t^l are underestimated for $t > t^*$. The more the latter (former) dominates, the more likely it is that Δ_t^l will fail when in fact social welfare is higher. ■

3. The PAGE model

To explore our theoretical arguments empirically, we employ the PAGE integrated assessment model (the 2002 version of the model, which is comprehensively described in Hope, 2006). For our present purposes, PAGE has two advantages. First, it is a stochastic model, capable of generating a probability distribution of consumption paths under climate change and climate-change policy. Second, each of its stochastic parameters is calibrated on the range of estimates available in the relevant literature, so it is capable of simulating a wide range of scenarios from small increases in consumption due to climate change to catastrophic decreases (i.e. driving per capita consumption down to subsistence levels), and from small negative costs of emissions reductions overall to large positive costs. PAGE uses relatively simple equations, but in focussing on uncertainty it approximates the results of a range of other

models, simple and complex. For example, its mean estimate of the marginal damage cost of carbon dioxide is close to the centre of a range of peer-reviewed studies (compare Hope, 2006 with Tol, 2005). Unless otherwise stated, we make no changes to the parameters reported in Hope (2006).

The primary features of the model are as follows:

- Ten uneven time periods spanning 200 years from 2001 to 2200, which focus computing resources on earlier periods in which projections of exogenous trends are more accurate, and eight large world-regions;
- Emissions of carbon dioxide and methane (including natural emissions stimulated by climate change in the model), as well as SF₆, and other greenhouse gases that contribute to background radiative forcing (e.g. NO₂ and (H)CFCs);
- The accumulation of anthropogenic greenhouse gas emissions in the atmosphere and resulting radiative forcing;
- Regional temperature increases arising from the difference between greenhouse warming and the regional cooling effect of sulphate aerosols. Atmospheric temperatures respond slowly to forcing as excess heat is transferred to land and ocean;
- Nonlinearity and transience in the damage caused by increases in regional mean temperature. Climate-change impacts in each analysis year are modelled as a polynomial function of regional mean temperature above a time-varying tolerable level of temperature change;
- Region-specific impacts of gradual climate change in two aggregate sectors – (1) ‘economic’ impacts on ‘market’ sectors of the economy like agriculture and energy and (2) ‘non-economic’, direct welfare impacts on ecosystems and human health;
- The possibility of future catastrophic impacts. When global mean temperature rises to a high level (an average of 5°C above its pre-industrial level), the chance of large losses in regional GDP in the range of 5-20% begins to appear. This chance increases by an average of about 10% per °C rise in global mean temperature beyond the mean trigger temperature of 5°C, so it is on average about 10% for a 6°C rise in global mean temperature above the pre-industrial level, c. 20% for a 7°C rise and so on;
- Adaptation to climate change, brought about by investment, that increases the time-varying tolerable level of climate change before economic losses (category (1) above) arise and reduces the intensity of impacts on both economic and non-economic sectors above the tolerable level. Adaptation is region-specific;

- Emissions-reduction costs that are determined by the depth of reduction below business-as-usual and are divided into low-cost and high-cost ranges. Emissions reductions are also specific to regions, as well as greenhouse gases.

There are 31 stochastic inputs to the model and we use a Latin Hypercube Sample to run 10,000 simulations, the maximum that is supported by the software. Latin Hypercube Sampling improves coverage of the full range of input values relative to a random or 'brute-force' Monte Carlo procedure, an especially advantageous feature when investigating probability distributions with a large skew (see below). We set a finite upper bound on the total costs of climate change (adaptation costs and residual impacts) and of emissions reductions at 99% of regional GDP, in order to ensure a finite expected shadow discount factor (see Weitzman, 2008).

Our exogenous scenario of economic growth, population growth and greenhouse gas emissions is taken from the Common Poles Image scenario of den Elzen *et al.* (2003), extrapolated from 2100 to 2200 by Alberth and Hope (2007). GDP growth averages 1.9% per annum over 200 years, being as much as 3.4% in 2100 and around 1.5-2% over the period 2100-2200. Population growth averages 0.2% over 200 years, falling from 1.2% in 2001 to virtually zero after 2100, when the global population peaks at around 10 billion. Thus GDP per capita grows at an average of 1.7% per annum over 200 years. We convert GDP to consumption with a constant exogenous savings rate of 20%. Annual emissions of CO₂ are around 38 gigatonnes (Gt) in 2100, peaking at around 57 Gt in 2060 and are constant at around 51 Gt from 2100 to 2200.

We consider two emissions-reduction scenarios, which aim to stabilise the atmospheric concentration of CO₂ at 550 parts per million (ppm) and 450 ppm respectively.⁶ Neither scenario minimises the cost of stabilisation; instead two sets of uniform emissions reductions are made across the developed world and the developing world. Minimising the cost of stabilisation would require a more regionally differentiated distribution of emissions reduction, as well as reductions in the emissions of all greenhouse gases, but these more simple scenarios are sufficient to illustrate our arguments empirically.

⁶ PAGE estimates that the atmospheric stock of CO₂ usually overshoots the target level due to natural emissions generated by warming.

4. Results

We begin by assessing the differences in shadow discount factors along the paths of utility per capita first described in section two. They are (i) the set of shadow discount factors along the ‘utopian’ path λ^u_t , (ii) the set of shadow discount factors along the path corresponding to business-as-usual climate change λ^b_t , and (iii) the set along the path corresponding to the project undertaken, $\lambda^{b+\Delta l}_t$. This provides us with a means of analysing whether the costs of climate change and the cost of emissions reductions constitute a non-marginal perturbation to the path of global per capita utility and will also pave the way to analyse the bias introduced by discounting the net welfare change from the investment Δl_t with either λ^u_t or λ^b_t .

We simply derive λ^u_t from the exogenous scenario of per capita consumption growth (see section three). This is of course unaffected by climate change and by emissions reductions. We estimate λ^b_t by using PAGE to estimate consumption per capita endogenously after deducting the costs of climate change (adaptation costs and residual impacts). We estimate $\lambda^{b+\Delta l}_t$ by running PAGE along a scenario in which the atmospheric stock of CO₂ is stabilised at 550 ppm. Consumption per capita is estimated after deducting both the costs of climate change and the cost of emissions reductions.

Tables 1 and 2 and figures 3a-d present time series of shadow discount factors estimated by PAGE for two parameterisations of the social welfare function described in equation (2), where the utility function is of the isoelastic form in equation (5). We describe the first parameterisation as ‘low’, setting the utility discount rate ρ to 0.1% per annum and the elasticity of social marginal utility of consumption η to unity (giving $u(c_t^i) = \ln c_t^i$). These are the same choices as were made in the integrated assessment modelling of chapter six of the *Stern Review* (Stern, 2007), for instance. For per capita consumption growth of, say, 2% per annum the shadow discount rate described in equation (6) would be 2.1%. However, it is important to once again emphasise that future per capita consumption growth is uncertain and, if climate change imposes very large costs, growth could be small or even negative. Starting with an exogenous scenario of per capita consumption growth of 2% per annum, the time-averaged, certainty-equivalent shadow discount rate net of the costs of climate change and of emissions reductions could thus be much lower than 2.1%, perhaps even negative. We describe the second parameterisation as ‘high’ and set ρ to 2% per annum and η to 2. This ‘duo’ of twos has been put forward by Weitzman (2007) as a plausible parameterisation of the shadow discount rate for

evaluation of climate policy. With per capita consumption growth of 2% per annum the shadow discount rate would be 6%. However, clearly other combinations of ρ and η could be used instead. Moreover, the same caveats apply with respect to uncertainty and the costs of climate change.

We also consider two different scenarios of climate change, using the important ‘climate sensitivity’ parameter. The climate sensitivity is the equilibrium change in global mean surface air temperature for a particular change in radiative forcing and it is commonly estimated in response to a doubling in the (equivalent) atmospheric stock of CO₂. Because the costs of climate change are a function of temperature increase, climate sensitivity is one of the primary determinants of the shadow value of emissions reductions (see Downing *et al.*, 2005). Our first scenario is termed ‘IPCC’ climate and employs a simple triangular probability distribution (minimum 1.5°C; mode 2.5°C; maximum 5°C) calibrated on the ‘likely’ range for the climate sensitivity suggested by the *Third Assessment Report* of the IPCC (IPCC, 2001) and modified only slightly by the *Fourth Assessment Report* (IPCC, 2007).⁷ Our second scenario employs a probability distribution with a large positive skew calibrated on Stainforth *et al.* (2005), which allows for very high values of the climate sensitivity with low probabilities. We calibrate a log-logistic function (minimum 1.6°C; mode 3.2°C; 99th percentile 8.2°C).⁸ Much attention in climate science and economics has recently focussed on estimated distributions with a large positive skew (see Roe and Baker, 2007, on the science and Weitzman, 2007, on the economics).

Table 1 reports time series of median shadow discount factors estimated by PAGE. We present median values first in order to emphasise later the role of low-probability/high-cost states of the world in the estimation of certainty-equivalent shadow discount factors and of certainty-equivalent net present values of emissions reductions. Table 1 shows that in all four scenarios the ‘utopian’ shadow discount factor λ^{u_t} (which by virtue of being exogenous to the model is not uncertain) is below the median shadow discount factor corresponding to the project undertaken $\lambda^{b+\Delta t_t}$ over the whole modelling horizon. This is consistent with figure 2, reflecting the underlying result that utility per capita is always higher in the ‘utopian’ case as there are no climate-change costs and no emissions-reduction

⁷ The ‘likely’ range in the IPCC *Third Assessment Report* is in fact 1.5-4.5°C, where ‘likely’ denotes an expert judgement of a likelihood greater than 66% but less than 90%. The IPCC *Fourth Assessment Report* concludes that the climate sensitivity “is likely to be in the range 2°C to 4.5°C with a best estimate of about 3°C, and is very unlikely [$<10\%$ probability] to be less than 1.5°C. Values substantially higher than 4.5°C cannot be excluded, but agreement of models with observations is not as good for those values” (IPCC, 2007, p12).

⁸ Where the log-logistic density function is described by $f(x) = \frac{\alpha t^{\alpha-1}}{\beta(1+t^\alpha)^2}$, $\alpha \approx 3.76$, $\beta \approx 1.96$.

costs. However, the differences are never especially large and our later results show that they have little effect on estimates of the net present value of emissions-reduction strategies.

The relative difference between λ^{u_t} and $\lambda^{b+\Delta t_t}$ is lowest in the scenario with ‘low’ welfare parameters and with the less sensitive ‘IPCC’ climate. λ^{u_t} dips just 3.8% below $\lambda^{b+\Delta t_t}$ in relative terms in 2200. This is a very small difference in absolute terms. In the scenario with ‘low’ welfare parameters but with the more sensitive ‘Stainforth’ climate the relative difference is higher, reaching 6.4% relative to $\lambda^{b+\Delta t_t}$ in 2200. This reflects the fact that the costs of climate change are higher in the ‘Stainforth’-climate scenario. The relative difference is also higher in the scenario with ‘high’ welfare parameters. The intuition behind this result might initially appear elusive, if we think of ρ and η purely in terms of intertemporal aggregation, as increases in both raise the shadow discount rate. Indeed, the same ρ discounts utility in every scenario, on every path and in every simulation run, so we would not expect it to have much effect on *relative* differences between the paths λ^{u_t} and $\lambda^{b+\Delta t_t}$. However, as has been emphasised in recent research on the welfare evaluation of climate policies (e.g. Dietz *et al.*, 2008), η has a triple role in studies where the costs of climate change and of emissions reductions are disaggregated by regions and by states of the world, as they are here. Across regions at any one point in time, higher η implies greater social inequality aversion, increasing the relative weight placed on changes in the consumption of poor regions brought about by climate change or by emissions reductions. With respect to states of the world, higher η implies greater risk aversion, which in turn implies greater concern about high-cost scenarios, which are assigned lower social welfare. The combined effect of greater social inequality aversion and greater risk aversion is to increase the difference in utility between the ‘utopian’ path and the path that accounts for the costs of climate change and for emissions reductions.

Table 1. Median shadow discount factors along different paths of utility per capita.

		'Low' welfare parameters					'High' welfare parameters				
		$\lambda^{b+\Delta l_t}$	λ^{u_t}	% diff. (relative to $\lambda^{b+\Delta l_t}$)	λ^{b_t}	% diff. (relative to $\lambda^{b+\Delta l_t}$)	$\lambda^{b+\Delta l_t}$	λ^{u_t}	% diff. (relative to $\lambda^{b+\Delta l_t}$)	λ^{b_t}	% diff. (relative to $\lambda^{b+\Delta l_t}$)
'IPCC' climate	2001	0.984827	0.982805	-0.20535	0.984829	0.000152	0.969875	0.965906	-0.40925	0.96988	0.000567
	2002	0.966244	0.963831	-0.2498	0.966245	8.28E-05	0.91697	0.912398	-0.49854	0.916973	0.000425
	2010	0.807365	0.804907	-0.3044	0.80695	-0.05134	0.554352	0.550982	-0.60793	0.553783	-0.10266
	2020	0.636402	0.634706	-0.26664	0.636148	-0.04004	0.287705	0.286167	-0.53447	0.287475	-0.07977
	2040	4.20E-01	4.19E-01	-0.38208	4.20E-01	-0.10698	8.76E-02	8.69E-02	-0.76724	8.74E-02	-0.21142
	2060	2.98E-01	2.96E-01	-0.7567	2.97E-01	-0.30116	3.07E-02	3.02E-02	-1.51751	3.05E-02	-0.60816
	2080	2.20E-01	2.18E-01	-1.06367	2.20E-01	-0.23891	1.17E-02	1.15E-02	-2.11071	1.17E-02	-0.47452
	2100	1.65E-01	1.62E-01	-1.46494	1.65E-01	-0.01998	4.56E-03	4.43E-03	-2.90012	4.56E-03	-0.03113
	2150	7.26E-02	7.06E-02	-2.68283	7.37E-02	1.473284	3.60E-04	3.41E-04	-5.26238	3.71E-04	2.8931
	2200	2.87E-02	2.76E-02	-3.75917	2.98E-02	4.088508	2.28E-05	2.12E-05	-7.3728	2.47E-05	8.175209
'Stainforth' climate	2001	0.984831	0.982805	-0.20579	0.984833	0.000142	0.969878	0.965906	-0.40959	0.969881	0.000309
	2002	0.96625	0.963831	-0.25041	0.966251	0.000135	0.916998	0.912398	-0.50158	0.917002	0.000436
	2010	0.807413	0.804907	-0.31031	0.806999	-0.05118	0.554433	0.550982	-0.62246	0.553857	-0.10402
	2020	0.636523	0.634706	-0.2855	0.636261	-0.04115	0.287806	0.286167	-0.56955	0.287568	-0.0828
	2040	4.21E-01	4.19E-01	-0.44577	4.20E-01	-0.10694	8.77E-02	8.69E-02	-0.88471	8.75E-02	-0.20474
	2060	2.98E-01	2.96E-01	-0.90194	2.97E-01	-0.29267	3.07E-02	3.02E-02	-1.79302	3.06E-02	-0.5791
	2080	2.21E-01	2.18E-01	-1.34616	2.21E-01	-0.18029	1.18E-02	1.15E-02	-2.6867	1.17E-02	-0.37662
	2100	1.66E-01	1.62E-01	-2.02416	1.66E-01	0.177201	4.61E-03	4.43E-03	-3.9671	4.63E-03	0.33856
	2150	7.37E-02	7.06E-02	-4.22177	7.57E-02	2.684433	3.72E-04	3.41E-04	-8.20844	3.92E-04	5.400679
	2200	2.95E-02	2.76E-02	-6.43309	3.13E-02	6.363765	2.41E-05	2.12E-05	-12.3807	2.73E-05	13.15551

Looking at the relationship between the business-as-usual shadow discount factors λ^{b_t} and the shadow discount factors corresponding to the project undertaken $\lambda^{b+\Delta l_t}$ we can see that the former is typically fractionally lower between 2001 and 2100, before rising above $\lambda^{b+\Delta l_t}$ for the remainder of the modelling horizon. This is also consistent with figure 2, indicating that the investment Δl_t to reduce CO₂ emissions initially depresses consumption below the level that would have been achieved if no emissions reductions had been made (i.e. producing net costs), before eventually crossing over and lifting consumption above the level under business-as-usual (i.e. yielding net benefits, by avoiding rising costs of climate change). Once again, the relative differences between the sets of shadow discount factors become more pronounced with ‘high’ values for the welfare parameters and in the ‘Stainforth’-climate scenario, for the same reasons given above. Similarly, however, they are not particularly large.

Table 2 and figures 3a-d report time series of certainty-equivalent (i.e. expected) shadow discount factors for λ^{b_t} and $\lambda^{b+\Delta l_t}$. In contrast to table 1, they show some significant relative differences between the sets of shadow discount factors. This illustrates the effect of low-probability/high-cost simulation runs, since the certainty equivalents for λ^{b_t} and $\lambda^{b+\Delta l_t}$ are now quite significantly different both from their comparable median values and from the set of ‘utopian’ shadow discount factors λ^{u_t} .

Again, the ‘utopian’ shadow discount factor λ^{u_t} is below the shadow discount factor corresponding to the project undertaken $\lambda^{b+\Delta l_t}$ over the whole modelling horizon. However the differences between the former and the certainty equivalent of the latter are now larger, reaching as much as 98.2% relative to $\lambda^{b+\Delta l_t}$ in 2200 (‘high’ welfare parameters, ‘Stainforth’ climate). With differences of these magnitudes, we would indeed expect to see some significant differences in estimates of the net present value of emissions reductions and our later results bear this out. In other words it appears that we are dealing with non-marginal perturbations.

Comparing λ^{b_t} with $\lambda^{b+\Delta l_t}$, large differences also emerge. As in table 1, λ^{b_t} is fractionally lower than $\lambda^{b+\Delta l_t}$ in the period up to 2100, before becoming higher in the period 2100-2200. In the scenario with ‘low’ welfare parameters and the ‘IPCC’ climate, λ^{b_t} rises to a modest 13.2% above $\lambda^{b+\Delta l_t}$ by 2200 (figure 3a). However, in the scenario with ‘high’ welfare parameters and the ‘IPCC’ climate, λ^{b_t} rises to as much as 368.6% above $\lambda^{b+\Delta l_t}$ by 2200 (figure 3b). This scenario amply illustrates the importance of low-probability/high-cost scenarios in the tails of the distribution of consumption losses, a topic which Martin Weitzman has recently been instrumental in bringing into focus (Weitzman, 2007). The

certainty-equivalent shadow discount factor on the path λ^{b_t} in 2200 is in fact far above the 95th percentile, indicating that just a handful of the 10,000 runs of the Latin Hypercube Sample are driving the certainty equivalent. The relative difference between λ^{b_t} and $\lambda^{b+\Delta I_t}$ is 36.6% in the scenario with ‘low’ welfare parameters and the ‘Stainforth’ climate in 2200 (figure 3c). With ‘high’ welfare parameters and under the ‘Stainforth’ climate, λ^{b_t} rises to over 290% above $\lambda^{b+\Delta I_t}$ by 2150 (figure 3d), before the gap closes to about 124% in 2200. In this latter scenario, the certainty-equivalent welfare costs of climate change are high with and without the investment in emissions reductions ΔI_t .⁹ Both certainty equivalents are above their respective 95th percentiles. The gap between λ^{b_t} and $\lambda^{b+\Delta I_t}$ widens and then closes because emissions reductions only succeed in postponing, not avoiding, spiralling costs of climate change in 2200.

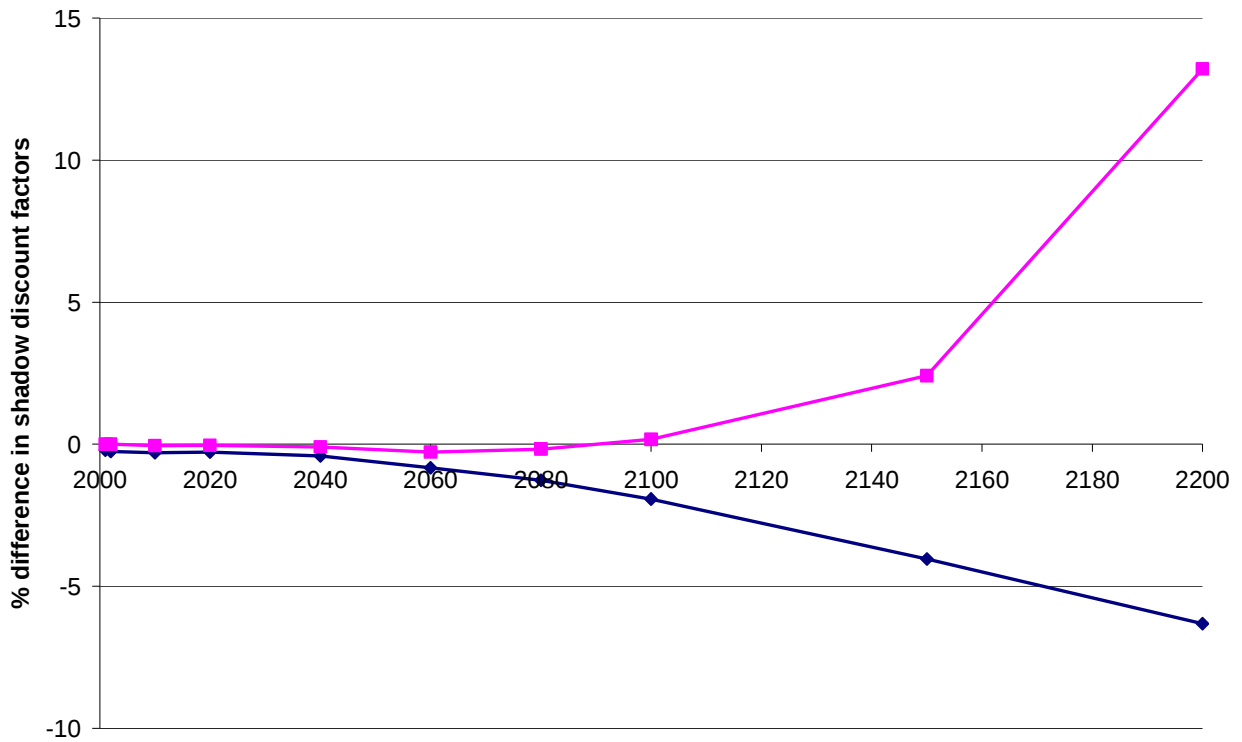
⁹ Indeed, both λ^{b_t} and $\lambda^{b+\Delta I_t}$ are in fact rising between 2080 and 2100, indicating that (weighted) global per capita consumption is falling for a time.

Table 2. Certainty-equivalent shadow discount factors along different paths of utility per capita.

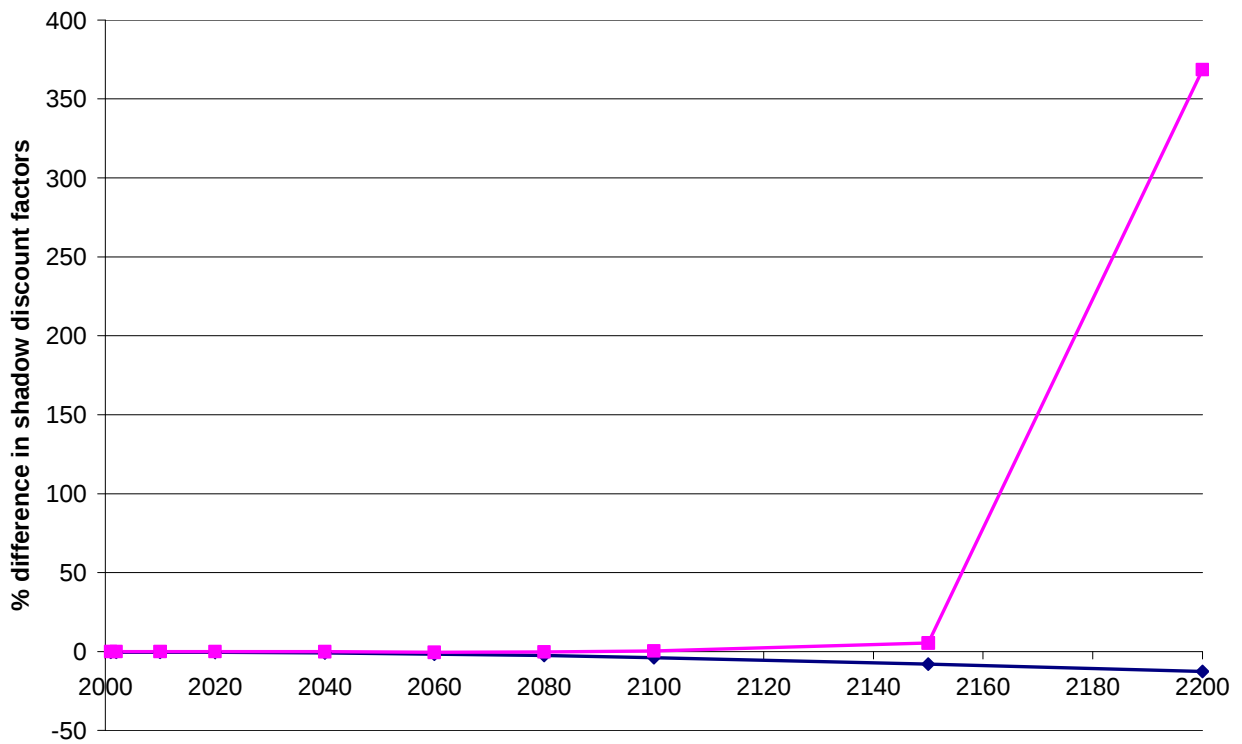
		'Low' welfare parameters					'High' welfare parameters				
		$\lambda^{b+\Delta l_t}$	λ^{u_t}	% diff. (relative to $\lambda^{b+\Delta l_t}$)	λ^{b_t}	% diff. (relative to $\lambda^{b+\Delta l_t}$)	$\lambda^{b+\Delta l_t}$	λ^{u_t}	% diff. (relative to $\lambda^{b+\Delta l_t}$)	λ^{b_t}	% diff. (relative to $\lambda^{b+\Delta l_t}$)
'IPCC' climate	2001	0.983986	0.982805	-0.2085	0.98486	0.000142	0.969949	0.965906	-0.416898	0.969952	0.000289
	2002	0.965343	0.963831	-0.25247	0.966273	0.000279	0.917026	0.912398	-0.504621	0.917031	0.000567
	2010	0.806251	0.804907	-0.30566	0.806979	-0.04901	0.554369	0.550982	-0.610857	0.553825	-0.097985
	2020	0.635652	0.634706	-0.27283	0.636208	-0.03681	0.287738	0.286167	-0.545983	0.287526	-0.073609
	2040	4.19E-01	0.41875	-0.40881	0.420068	-0.09523	8.76E-02	8.69E-02	-0.819664	8.75E-02	-0.19059
	2060	0.29653	0.2955	-0.82947	0.297158	-0.27331	3.07E-02	3.02E-02	-1.659477	3.05E-02	-0.545461
	2080	0.219034	0.217951	-1.2709	0.220373	-0.17377	1.18E-02	1.15E-02	-2.549986	1.17E-02	-0.339136
	2100	0.163241	0.162223	-1.93017	0.165697	0.169935	4.61E-03	4.43E-03	-3.887266	4.62E-03	0.358588
	2150	7.13E-02	7.06E-02	-4.04167	7.54E-02	2.4086	3.72E-04	3.41E-04	-8.1359	3.91E-04	5.23291
	2200	2.79E-02	2.76E-02	-6.32202	3.33E-02	13.21191	2.42E-05	2.12E-05	-12.75904	1.14E-04	368.6301
'Stainforth' climate	2001	0.984865	0.982805	-0.20915	0.984866	0.000132	0.969962	0.965906	-0.418243	0.969965	0.000278
	2002	0.96628	0.963831	-0.25353	0.966283	0.000269	0.917046	0.912398	-0.506812	0.917051	0.000545
	2010	0.807436	0.804907	-0.31319	0.80704	-0.04903	0.554456	0.550982	-0.626578	0.553912	-0.098114
	2020	0.636597	0.634706	-0.29714	0.636361	-0.03712	0.287889	0.286167	-0.598043	0.287675	-0.074369
	2040	0.420941	0.41875	-0.52067	0.420539	-0.09555	8.86E-02	8.69E-02	-1.93685	8.84E-02	-0.262399
	2060	0.298972	0.2955	-1.16128	0.298222	-0.25086	6.12E-02	3.02E-02	-50.63317	6.10E-02	-0.239371
	2080	0.224713	0.217951	-3.0093	0.224637	-0.03373	3.51E-02	1.15E-02	-67.36364	3.68E-02	4.768995
	2100	0.178318	0.162223	-9.02581	0.175238	-1.72681	5.35E-02	4.43E-03	-91.72768	5.42E-02	1.278533
	2150	9.72E-02	7.06E-02	-27.3376	0.111416	14.61194	9.99E-03	3.41E-04	-96.58274	3.90E-02	290.7703
	2200	5.05E-02	0.027574	-45.4242	6.90E-02	36.63914	1.20E-03	2.12E-05	-98.24415	2.70E-03	124.325

Figure 3. Difference in shadow discount factors as a percentage of $\lambda^{b+\Delta l_t}$. λ^{u_t} denoted by diamonds, λ^{b_t} by squares. λ^{b_t} and $\lambda^{b+\Delta l_t}$ are certainty equivalents.

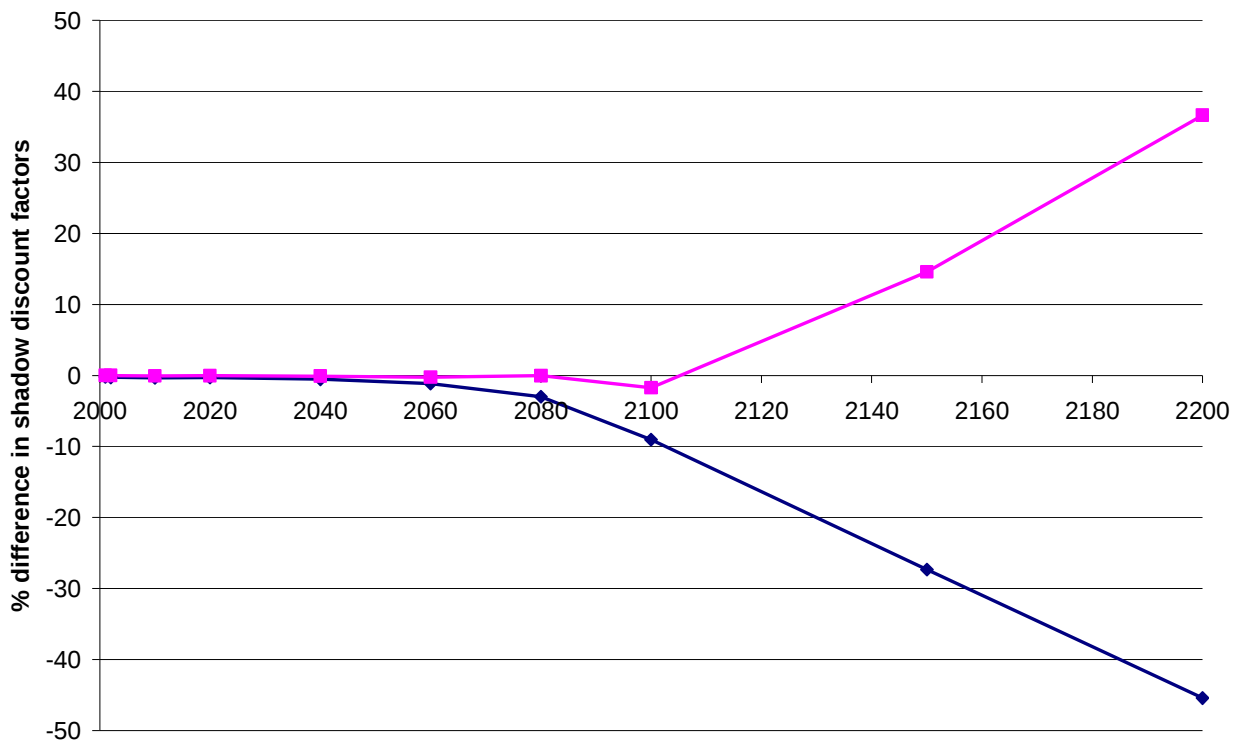
a. Low welfare parameters, IPCC climate.



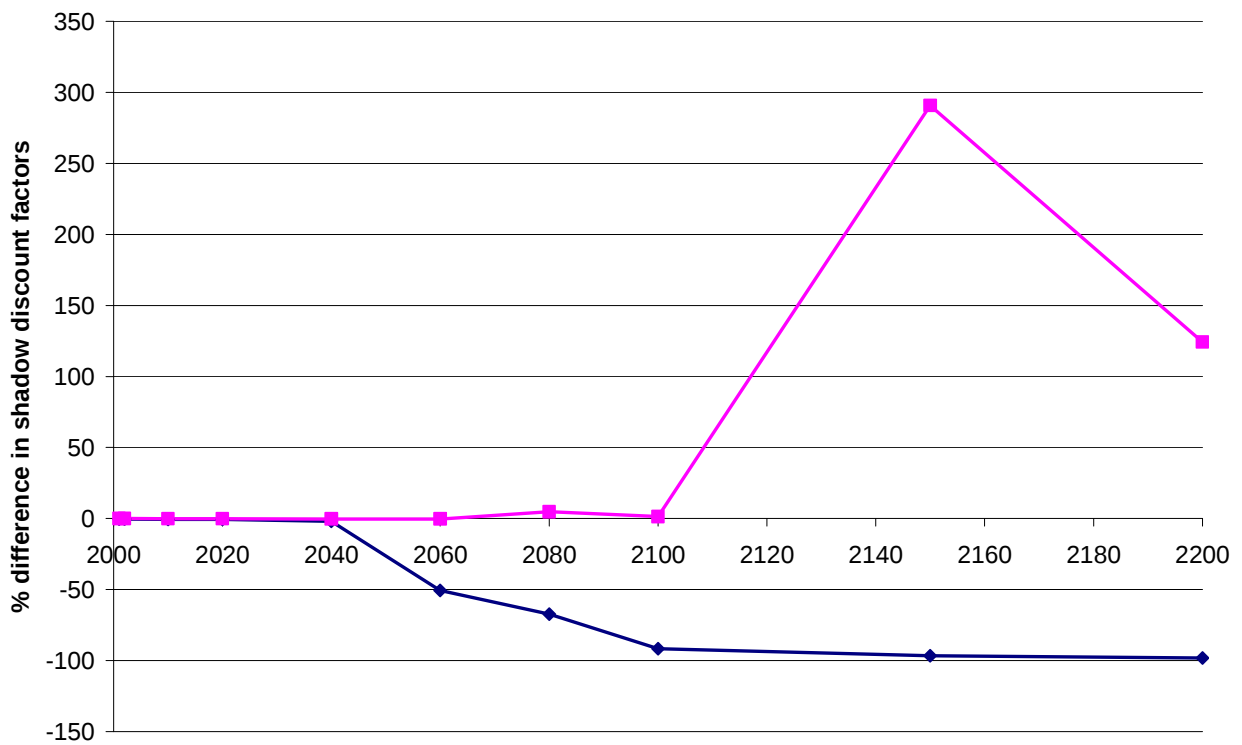
b. High welfare parameters, IPCC climate.



c. Low welfare parameters, Stainforth climate.



d. High welfare parameters, Stainforth climate.



We now move on to test propositions 2a and 2b empirically. Tables 3-6 report estimates of the net present value (hereafter NPV) of two investments to stabilise the atmospheric stock of CO₂. The first investment, considered above, stabilises the atmospheric stock of CO₂ at 550 ppm, while we now consider a second more ambitious investment that stabilises the stock at 450 ppm. These are our two ‘large projects’ Δ^l_t and we evaluate them along the three sets of shadow discount factors introduced conceptually in section two and empirically at the beginning of this section. To recap, the NPV of a non-marginal project Δ^l_t can be estimated by taking the difference between welfare changes with the project and without the project (i.e. following business-as-usual). The former are evaluated with $\lambda^{b+\Delta^l_t}$, the latter with λ^{b_t} . For convenience we label this the ‘non-marginal method’. We cannot use the simplification in equation (7), because it discounts changes in per capita consumption on both paths with the same set of shadow discount factors λ^{i_t} , although for a non-marginal investment this set cannot simultaneously be consistent both with $u(c^{b_t+\Delta^l_t})$ and with $u(c^{b_t})$. It may not be consistent with either, as with λ^{u_t} . For convenience we label this the ‘marginal method’. We are interested in the sign and size of the bias caused by estimating the NPV of the project using the marginal method, either discounting with λ^{u_t} or λ^{b_t} , especially instances in which the bias leads Δ^l_t to fail (pass) the cost-benefit test using the marginal method when in fact social welfare is higher (lower), as estimated by the non-marginal method.

Table 3 reports median estimates of NPV for the ‘IPCC’-climate scenario, while table 4 does so for the ‘Stainforth’-climate scenario. In both tables, we disaggregate the parameterisation of the social welfare function to consider four combinations of ρ and η : 0.1% per annum and unity; 2% p.a. and unity; 0.1% p.a. and 2; 2% p.a. and 2. Looking at both tables 3 and 4, we can see that the differences in median estimates of NPV caused by discounting with marginal and non-marginal methods are small. We would have expected this given the small relative and especially absolute differences between the different sets of *median* shadow discount factors outlined above. In particular, in no case is stabilisation at either 550 ppm or 450 ppm estimated to produce positive (negative) NPV with the non-marginal method, while producing negative (positive) NPV when estimated with the marginal method, either discounting on λ^{u_t} or λ^{b_t} . Put another way, we find no instances in which, using median estimates, the investment Δ^l_t passes (fails) a cost-benefit test when discounted with the non-marginal method, while failing (passing) when discounted with the marginal method, be that on λ^{u_t} or λ^{b_t} .

Where ρ is 0.1% p.a., Δ^l_t always yields positive NPV. NPV is highest when the ‘project’ is discounted with the non-marginal method. Stabilising the atmospheric stock of CO₂ at 550 ppm under the ‘IPCC’-

climate scenario and where ρ is 0.1% p.a. and η is unity, median NVP is estimated to be around \$25 trillion by the non-marginal method, while it is estimated to be around \$24 trillion and \$22 trillion by the marginal method, discounting with λ^{b_t} and λ^{u_t} respectively. Across all the scenarios in which the non-marginal method estimates positive NPV, this pattern is reproduced. NPV is underestimated most by discounting the net benefits of Δ^l_t with λ^{u_t} . Discounting with λ^{b_t} also underestimates NPV where it is positive, but by less. In relation to propositions 2a and 2b, it shows that the amount by which the net benefits of Δ^l_t are underestimated after t^* , which table 2 indicates lies in the interval 2060-2100, exceeds the amount by which the net costs of Δ^l_t are underestimated before t^* . In a minority of cases where with only one exception ρ is 2% p.a. and η is 2, Δ^l_t yields negative NPV. In these cases, discounting with the marginal method either underestimates or overestimates the welfare loss due to Δ^l_t .

Table 3. Median NPV estimates (trillion year 2000 US\$) for stabilisation of the atmospheric stock of CO₂ at 550 ppm and 450 ppm from marginal and non-marginal discounting, 'IPCC' climate.

Stabilisation scenario	Welfare parameters		Marginal method		Non-marginal method, $\lambda^{b+\Delta^l_t}$ and λ^{b_t}
	ρ	η	λ^{u_t}	λ^{b_t}	
550 ppm	0.1	1	21.98085	23.95097	25.06496
	2	1	0.2221948	0.3081444	0.3495508
	0.1	2	1.250756	1.399175	1.478104
	2	2	-0.2475755	-0.2366915	-0.2322771
450 ppm	0.1	1	3.273688	3.78062	3.876477
	2	1	-1.65565	-1.493194	-1.511884
	0.1	2	0.2506965	0.4215347	0.3652578
	2	2	-1.291581	-1.282646	-1.360151

Table 4. Median NPV estimates (trillion year 2000 US\$) for stabilisation of the atmospheric stock of CO₂ at 550 ppm and 450 ppm from marginal and non-marginal discounting, ‘Stainforth’ climate.

Stabilisation scenario	Welfare parameters		Marginal method		Non-marginal method, $\lambda^{b+\Delta l_t}$ and λ^{b_t}
	ρ	η	λ^{u_t}	λ^{b_t}	
550 ppm	0.1	1	39.38449	44.87679	48.25996
	2	1	1.160723	1.389085	1.519899
	0.1	2	2.234065	2.590041	2.818675
	2	2	-0.1205113	-8.709299E-02	-6.662351E-02
450 ppm	0.1	1	56.43198	63.81946	67.12626
	2	1	0.1918111	0.5455124	0.6141078
	0.1	2	2.496403	3.005591	3.065243
	2	2	-0.9639093	-0.910376	-0.9657329

Tables 5 and 6 report 5th percentile, mean (i.e. certainty-equivalent) and 95th percentile estimates of NPV for the ‘IPCC’ and ‘Stainforth’ climates respectively. We can see that there are now substantial differences between the certainty-equivalent NPV estimates from the marginal and non-marginal methods. Moreover, where ρ is 2% per annum, Δl_t passes the cost-benefit test with the non-marginal method but sometimes fails with the marginal method, discounting with λ^{u_t} . Take for instance the estimates of the NPV of stabilisation at 450 ppm, where $\rho=2\%$ p.a., η is unity and the climate scenario is ‘IPCC’. The mean NPV of the project is -\$1 trillion discounted with λ^{u_t} , \$0.5 trillion discounted with λ^{b_t} and \$1.6 trillion discounted with the non-marginal method. So Δl_t increases social welfare and should pass the cost-benefit test, but in fact it fails when evaluated with the marginal method, discounting with λ^{u_t} . As before, the extent to which the net benefits of Δl_t are underestimated after t^* exceeds the extent to which the net costs are underestimated before t^* .

Similarly, for stabilisation at 550 ppm CO₂ under the ‘Stainforth’ climate scenario and where ρ is 2% p.a. and η is 2, Δl_t passes the cost-benefit test with the non-marginal method (mean NPV=\$525.9 trillion) and with the marginal method where we use λ^{b_t} (mean NPV=\$411.8 trillion), but fails with the marginal method where we use λ^{u_t} (mean NPV equals \$-0.04 trillion). This latter case is rather more interesting in the respect that the certainty-equivalent estimate of NPV lies beyond the 95th percentile, both where the investment is estimated with the non-marginal method, and where it is estimated with the marginal method, discounting with λ^{b_t} . This again illustrates the role of low-probability/high-cost scenarios in the tails of the distribution of consumption losses, with a likelihood of less than 2.5%. Indeed, looking across the suite of scenarios, this occurs frequently, especially under the more sensitive ‘Stainforth’ climate, where η is 2, and especially where these two are combined.

The estimates clearly suggest that for plausible parameter values proposition 1 does indeed give a misleading signal of the welfare value of the large investment in emissions reductions Δ^l_t . In particular, discounting with λ^{u_t} frequently leads emissions reductions to fail the cost-benefit test when in fact they should pass. The investment Δ^l_t yields positive NPV – passing the cost-benefit test – in all 16 scenarios considered, yet discounting with λ^{u_t} it passes in only 11 cases. This is of concern since many analyses proceed to use λ^{u_t} as the shadow discount rate for evaluation of climate-change policies.

5. Conclusions

The impacts of climate change could be large and irreversible, so that choices today over how much to reduce greenhouse gas emissions now and in the future could lead to very different outcomes in terms of future growth. In addition, what growth we might expect under business-as-usual climate change and under a policy to stabilise the atmospheric stock of greenhouse gases may be very different to what we would extrapolate from past trends, when there was little climate change and few emissions reductions. This is an immediate problem for the simplified technique of cost-benefit analysis developed to appraise small projects, which assumes such projects are but infinitesimal perturbations around the growth path. While in the last ten years or so we have seen significant developments in the theory and practice of discounting under uncertainty about the future, this uncertainty is not of the same kind as posed by the present problem. These approaches consider exogenous uncertainty, whereas climate-policy choices generate endogenous uncertainty.

In this paper we have essentially asked the question; how wrong can we be if we appraise global climate strategies using marginal techniques? We have shown that in theory we could fail strategies to stabilise the atmospheric stock of greenhouse gases using marginal techniques, when in fact they should pass as social welfare is increased, and vice versa. More importantly, we use an integrated assessment model to identify some plausible empirical scenarios in which simplified cost-benefit analysis fails stabilisation of the atmospheric stock of CO₂ at 550 ppm and 450 ppm, when in fact the strategies should have passed.

Table 5. NPV estimates (trillion year 2000 US\$) for stabilisation of the atmospheric stock of CO₂ at 550 ppm and 450 ppm from marginal and non-marginal discounting, 'IPCC' climate. Investments that pass a cost-benefit test are shaded with green vertical lines.

Stabilisation scenario	Welfare parameters		Marginal method						Non-marginal method, $\lambda^{b+\Delta l_t}$ and λ^{b_t}		
			λ^{u_t}			λ^{b_t}			λ^{b_t}		
	ρ	η	5%	mean	95%	5%	mean	95%	5%	mean	95%
550 ppm	0.1	1	-4.614307	32.24886	101.5625	-4.53425	73.86954	135.3216	-4.53213	121.0873	161.4813
	2	1	-2.612756	0.5600098	4.885489	-2.607378	1.451301	6.157797	-2.63298	2.424944	7.179305
	0.1	2	-2.375469	1.486264	7.25689	-2.389154	35.29485	9.469227	-2.423009	84.48347	11.31253
	2	2	-1.089686	-0.236225	0.6385897	-1.094638	1.347219	0.7234523	-1.113638	3.747468	0.798826
450 ppm	0.1	1	-9.774612	7.182485	36.83235	-9.73973	21.38427	48.71479	-10.08039	31.041	55.0088
	2	1	-7.791039	-1.045492	7.776452	-7.791267	0.5087322	9.664615	-8.088624	1.55526	10.80208
	0.1	2	-5.678893	1.228527	11.51732	-5.708972	51.28444	15.04392	-6.204973	93.31457	17.04997
	2	2	-3.411829	-1.260324	0.9223793	-3.436602	13.247	1.078343	-3.745891	23.00665	1.131201

Table 6. NPV estimates (trillion year 2000 US\$) for stabilisation of the atmospheric stock of CO₂ at 550 ppm and 450 ppm from marginal and non-marginal discounting, 'Stainforth' climate. Investments that pass a cost-benefit test are shaded with green vertical lines.

Stabilisation scenario	Welfare parameters		Marginal method						Non-marginal method, $\lambda^{b+\Delta l_t}$ and λ^{b_t}		
			λ^{u_t}			λ^{b_t}			λ^{b_t}		
	ρ	η	5%	mean	95%	5%	mean	95%	5%	mean	95%
550 ppm	0.1	1	0.9382894	56.58527	157.7106	1.187505	773.1768	346.1083	1.296425	1259.05	579.6542
	2	1	-2.117831	2.036328	8.167101	-2.096128	31.64362	14.51526	-2.112934	53.81863	21.04856
	0.1	2	-1.751392	3.496637	11.84723	-1.740306	2227.594	25.05553	-1.763594	4006.189	40.15265
	2	2	-1.02076	-0.041029	0.9176099	-1.026384	411.8289	1.554858	-1.046949	525.9126	2.153532
450 ppm	0.1	1	-1.977626	82.86451	243.3554	-1.586845	852.8906	496.0611	-1.733245	1315.252	692.0385
	2	1	-6.717455	2.143417	14.52015	-6.686595	72.14921	23.59795	-6.951128	93.60942	30.39329
	0.1	2	-4.631618	4.774298	19.49258	-4.646131	14438.25	38.3318	-5.059438	24683.64	55.59065
	2	2	-3.238635	-0.851896	1.712767	-3.261966	85018.15	2.753375	-3.579923	161393.4	3.38051

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